

# 几何分析研讨会（武汉大学 2016）

Workshop on Geometric Analysis ( Wuhan University 2016 )

## 会议手册

时间： 2016 年 12 月 3 日 -5 日

地点： 武汉大学数学与统计学院



*Workshop on Geometric Analysis*

## 几何分析研讨会

(武汉大学 2016)

邀请报告人:

|              |              |
|--------------|--------------|
| 陈大广 (清华大学)   | 程亮 (华中师范大学)  |
| 丁琪 (复旦大学)    | 韩小利 (清华大学)   |
| 华波波 (复旦大学)   | 李宇翔 (清华大学)   |
| 刘世平 (中国科技大学) | 罗勇 (武汉大学)    |
| 毛井 (湖北大学)    | 邱红兵 (武汉大学)   |
| 王作勤 (中国科技大学) | 魏国新 (华南师范大学) |
| 夏超 (厦门大学)    | 杨翎 (复旦大学)    |
| 张会春 (中山大学)   | 张永兵 (中国科技大学) |
| 朱苗苗 (上海交通大学) |              |

组织者: 陈化、陈群、李光汉、邱红兵、徐旭

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资助: 国家自然科学基金

武汉大学数学与统计学院

武汉大学数学协同创新中心

时间: 2016年12月3-5日

地点: 武汉大学数学与统计学院



## 会议议程

报到： 12月2日，10:00–20:00，地点：武汉弘毅大酒店

12月3日，08:00–08:40，地点：武汉大学数学与统计学院数学会堂

| 12月3日，星期六，上午 (主持人: 陈群) |                  |     |
|------------------------|------------------|-----|
| 08:40 – 09:10          | 开幕式 (陈化院长致辞) 及合影 |     |
| 09:10 – 09:55          | 李宇翔              |     |
| 09:55 – 10:40          | 杨 翎              |     |
| 10:40 – 11:00          | 茶 歇              |     |
| 11:00 – 11:45          | 魏国新              |     |
| 12:00 – 14:00          | 午餐 (珞珈山庄)        |     |
| 12月3日，星期六，下午           |                  |     |
| 14:00 – 14:45          | 主持人:<br>李宇翔      | 丁 琦 |
| 14:45 – 15:30          |                  | 韩小利 |
| 15:30 – 15:50          | 茶 歇              |     |
| 15:50 – 16:35          | 主持人:<br>张永兵      | 邱红兵 |
| 16:35 – 17:20          |                  | 张会春 |
| 18:00 – 19:30          | 晚宴 (江南小观园)       |     |

| 12月4日，星期日，上午 (主持人: 韩小利) |             |     |
|-------------------------|-------------|-----|
| 09:00 – 09:45           | 夏 超         |     |
| 09:45 – 10:30           | 王作勤         |     |
| 10:30 – 10:50           | 茶 歇         |     |
| 10:50 – 11:35           | 陈大广         |     |
| 12:00 – 14:00           | 午餐 (珞珈山庄)   |     |
| 12月4日，星期日，下午            |             |     |
| 14:00 – 14:45           | 主持人:<br>刘世平 | 朱苗苗 |
| 14:45 – 15:30           |             | 张永兵 |
| 15:30 – 15:50           | 茶 歇         |     |
| 15:50 – 16:35           | 主持人:<br>朱安强 | 程 亮 |
| 16:35 – 17:20           |             | 毛 井 |
| 18:00 – 19:00           | 晚餐 (珞珈山庄)   |     |

| 12月5日，星期一，上午 (主持人: 李光汉) |           |  |
|-------------------------|-----------|--|
| 09:00 – 09:45           | 华波波       |  |
| 09:45 – 10:30           | 罗 勇       |  |
| 10:30 – 10:50           | 茶 歇       |  |
| 10:50 – 11:35           | 刘世平       |  |
|                         | 会议结束      |  |
| 12:00 – 13:00           | 午餐 (珞珈山庄) |  |

## 会议具体议程

**报到:** 12月2日, 10:00-20:00, 地点: 武汉弘毅大酒店

12月3日, 08:00-08:40, 地点: 武汉大学数学与统计学院数学会堂

**会议地点:** 武汉大学数学与统计学院数学会堂

### 12月3日, 星期六, 上午

\*8:00 从弘毅酒店乘车往武大数院

**主持人: 陈群**

08:40 – 09:10 开幕式 (陈化院长致辞) 及合影

09:10 – 09:55 李宇翔

Metrics in a fixed conformal class with bounded measure and  $\|R\|_{L^p}$

09:55 – 10:40 杨翎

On Lawson-Osserman Constructions

10:40 – 11:00 茶歇

11:00 – 11:45 魏国新

Complete  $\lambda$ -hypersurfaces

12:00 – 14:00 午餐 (珞珈山庄)

### 12月3日, 星期六, 下午

**主持人: 李宇翔**

14:00 – 14:45 陈大广

Bounds for ratios of the membrane eigenvalues

14:45 – 15:30 韩小利

映射洛伦茨流形的调和映射的热流

15:30 – 15:50 茶歇

**主持人: 张永兵**

15:50 – 16:35 邱红兵

V-harmonic maps and its geometric applications

16:35 – 17:20 张会春

Heat equations on non-smooth metric measure spaces with generalized Ricci lower bounds

18:00 – 19:30 晚宴 (江南小观园)

\*19:30 从武汉大学大门乘车往弘毅酒店

### 12月4日, 星期天, 上午

\*8:20 从弘毅酒店乘车往武大数院

**主持人: 韩小利**

|               |           |   |
|---------------|-----------|---|
| 09:00 – 09:45 | 夏 超       | Generalized Reilly type formulas and applications on geometric inequalities |
| 09:45 – 10:30 | 王作勤       | On the Isospectral Compactness for 4-manifolds                              |
| 10:30 – 10:50 | 茶 歆       |   |
| 10:50 – 11:35 | 丁 琦       | Translating solitons of mean curvature flow                                 |
| 12:00 – 14:00 | 午餐 (珞珈山庄) |   |

**12月4日, 星期天, 下午**

**主持人: 刘世平**

|               |     |  |
|---------------|-----|--|
| 14:00 – 14:45 | 朱苗苗 | Blow-up analysis at the boundary for approximate harmonic maps from surfaces |
| 14:45 – 15:30 | 张永兵 | Lagrangian F-stability of closed Lagrangian self-shrinkers                   |
| 15:30 – 15:50 | 茶 歆 |  |

**主持人: 朱安强**

|               |              |  |
|---------------|--------------|--|
| 15:50 – 16:35 | 程 亮          | On the long-time solutions of mean curvature flows |
| 16:35 – 17:20 | 毛 井          | The BCN-conjecture on hyperbolic surfaces          |
| 18:00 – 19:00 | 晚餐 (珞珈山庄)    |  |
| *19:00        | 从珞珈山庄乘车往弘毅酒店 |  |

**12月5日, 星期一, 上午**

**主持人: 李光汉**

|               |              |  |
|---------------|--------------|--|
| 09:00 – 09:45 | 华波波          | Combinatorial curvature of a planar graph                                |
| 09:45 – 10:30 | 罗 勇          | On Willmore Legendrian surfaces in $S^5$ and csL Willmore surfaces       |
| 10:30 – 10:50 | 茶 歆          |  |
| 10:50 – 11:35 | 刘世平          | Cheeger constant, spectral clustering and eigenvalue ratios of Laplacian |
|               |              | <b>会议结束</b>  |
| 12:00 – 13:00 | 午餐 (珞珈山庄)    |  |
| *13:00        | 从珞珈山庄乘车往弘毅酒店 |  |

注: \* 仅适用于在弘毅酒店住宿的与会者

## 报告题目与摘要

报告人：陈大广

题 目：Bounds for ratios of the membrane eigenvalues

摘要：In this talk, we will talk about the recent developments of the bounds for ratios of the membrane eigenvalues. For a bounded planar region in  $\mathbb{R}^2$ , we obtain the ratios of lower order eigenvalues of Laplace operator. Combining our results with the recursive formula for eigenvalue, we can obtain better upper bound of the  $(k+1)$ -th ( $k \geq 3$ ) membrane eigenvalues. This work is partly joint work with Dr. T. Zheng.

报告人：程亮

题 目：On the long-time solutions of mean curvature flows

摘要：In this talk, we will first recall the singularity models of long-time solutions of mean curvature flows which were introduced by Hamilton. We next show that the mean curvature flow for entire graphs satisfying certain conditions could be Type IIb. Our results lead us to getting the nontrivial examples for the Type IIb mean curvature flow. We also introduce the monotonicity formulas for the mean curvature flow which are related to self-expanders. Using the monotonicity formulas, we can show the Type III mean curvatures asymptotically like self-expanders under certain conditions. This is the joint work with Natasa Sesum.

报告人：丁琪

题 目：Translating solitons of mean curvature flow.

摘要：In this talk, I shall discuss the existence and rigidity for translating solitons of mean curvature flow in both Euclidean space and Minkowski space.

报告人：韩小利

题 目：映到洛伦茨流形的调和映射的热流

摘要：我们研究从紧黎曼曲面映到洛伦茨流形的调和映射，把通常的热流改进为一种新的椭圆抛物流来克服洛伦茨度量带来的困难。一方面，通过假设适当的几何条件，我们证明

了热流的全局存在性，这蕴含了给定的同伦类中调和映射的存在性。另一方面，我们还证明了这种调和映射序列的能量等式。

报告人：华波波

题 目：Combinatorial curvature of a planar graph

摘 要：Combinatorial curvature of a planar graph is defined as a discrete analogue to Gaussian curvature of a Riemannian surface. We are interested in the structure of infinite graphs with nonnegative combinatorial curvature. In this talk, we will discuss some related problems and results. This is a joint work with Su Yanhui.

报告人：李宇翔

题 目：Metrics in a fixed conformal class with bounded measure and  $\|R\|_{L^p}$

摘 要：Let  $(M,g)$  be a smooth compact Riemannian manifold without boundary. Let  $g_k = u_k^{4/(n-2)} g$ , and  $R_k$  be the constant curvature of  $g_k$ . We assume

$$\text{vol}(M, g_k) + \|R_k\|_{L^p} < C, \text{ where } p > \frac{n}{2}.$$

We will use 3-circle theorem to study the convergence of  $g_k$ .

报告人：刘世平

题 目：Cheeger constant, spectral clustering and eigenvalue ratios of Laplacian

摘 要：We will explain an optimal eigenvalue ratio estimate on closed Riemannian manifolds with nonnegative curvature. The method is inspired by analysis on discrete graphs involving Cheeger type constant. In the end, we will discuss applications in both discrete and continuous setting.

报告人：罗 勇

题 目：On Willmore Legendrian surfaces in  $S^5$  and csL Willmore surfaces

摘 要：In this talk we will give a classification result for Willmore Legendrian spheres in  $S^5$  and a gap theorem for Willmore Legendrian surfaces in  $S^5$  is provided, which generalizes a related gap theorem for minimal Legendrian surfaces in  $S^5$ . We will also introduce a geometrically constrained variation problem for the Willmore functional of surfaces in  $S^5$ .

We introduce a flow method aiming to prove the existence and as a first step, prove the well posedness of this flow.

报告人：毛 井

题 目：The BCN-conjecture on hyperbolic surfaces

摘 要：In this talk, we give a partial answer to the Berestycki –Caffarelli – Nirenberg conjecture on hyperbolic surfaces, which is based on a joint-work with Prof. José Espinar.

报告人：邱红兵

题 目：V-harmonic maps and its geometric applications

摘 要：In this talk, we give an existence theorem for V-harmonic maps from complete manifolds into regular balls. We observe that the Gauss maps of self-shrinkers and translating solitons are both some V-harmonic maps, we consider some geometric applications such as rigidity theorems. Meanwhile, we can also obtain an improved Liouville type theorem.

报告人：王作勤

题 目：On the Isospectral Compactness for 4-manifolds

摘 要：It is well known that the spectrum of the Laplace-Beltrami operator on a compact Riemannian manifold is quite rigid. People suspect that on any compact manifold, the set of Riemannian metrics that are isospectral to any given metric is compact under the  $C^{\infty}$  topology. However, this is only proven in the case of surfaces by Osgood-Phillips-Sarnak. Following works of Chang-Yang for 3-manifolds and of Chen-Xu for 4-manifolds, we will prove that for certain classes of Riemannian metrics on 4-manifolds, the isospectral metrics in the same conformal class form a compact set. This is a joint work with Xianfu Liu.

报告人：魏国新

题 目：Complete  $\lambda$ -hypersurfaces

摘 要：We prove that  $\lambda$ -hypersurface is a critical point of the weighted area

functional for the weighted volume-preserving variations, give classifications of complete  $\lambda$ -hypersurfaces with  $H-\lambda \geq 0$  and study properties of  $\lambda$ -hypersurfaces. This is a joint work with Professor Qing-Ming Cheng.

报告人：夏 超

题 目：Generalized Reilly type formulas and applications on geometric inequalities

摘要：Reilly's formula is the integral version of Bochner's formula for manifolds with boundary. It has numerous applications when Ricci curvature is nonnegative. In this talk, I will present two kinds of generalized Reilly type formulas for manifolds with boundary which are applicable for manifolds satisfying either a sectional curvature lower bound or a sub-static condition. In particular, we use it to reprove Brendle's result on Heintze-Karcher type inequality for warped product manifolds. Moreover, we give several new geometric inequalities using these formulas. The talk is a report of joint works with Guohuan Qiu, and separately with Junfang Li.

报告人：杨 翎

题 目：On Lawson-Osserman Constructions

摘要：Lawson-Osserman constructed three types of non-parametric minimal cones of higher codimension based on Hopf fibrations between Euclidean spheres, which can be seen as Lipschitz solutions to the minimal surface equations which are not differentiable, thereby making sharp contrast to the regularity theorem for minimal hypersurfaces in Euclidean spaces. In this paper, the above constructions are generalized in a more general scheme. Once a mapping  $f$  can be written as the composition of a Riemannian submersion from a Euclidean sphere and an isometric minimal immersion into another Euclidean sphere, the graph of  $f$  yields a non-parametric minimal cone. Because the choices of the second component form huge moduli spaces, our constructions produce a constellation of uncountable many examples. For each such cone, there exists an entire minimal graph whose tangent cone at infinity is just the given one. Moreover, surprising phenomena on the existence, non-uniqueness and non-minimizing for the Dirichlet problem are discovered, due to the amusing spiral asymptotic behaviour of a particular autonomous system on the 2-

plane.

报告人：张会春

题 目：Heat equations on non-smooth metric measure spaces with generalized Ricci lower bounds.

摘要：In this talk, we will introduce some developments about geometric analysis on singular metric measure spaces with the Riemannian curvature-dimension condition, which is a synthetic notion for lower Ricci curvature bounds. In particular, we will introduce a sharp Yau's gradient estimate for harmonic functions, lower bounds for the first eigenvalue of Laplacian, and a local Li-Yau's estimate for the heat equations. This talk is based on a joint work with Xi-Ping Zhu, and a joint work with Yin Jiang.

报告人：张永兵

题 目：Lagrangian F-stability of closed Lagrangian self-shrinkers

摘要：We will talk about the Lagrangian F-stability of closed Lagrangian self-shrinkers and the implication about the generic singularities of the Lagrangian mean curvature flow. This is a joint work with Jiayu Li.

报告人：朱苗苗

题 目：Blow-up analysis at the boundary for approximate harmonic maps from surfaces

摘要：In this talk, we shall discuss the blow-up analysis at the free boundary for approximate harmonic maps from surfaces with applications to the two dimensional harmonic map flow with free boundary. Also, we shall briefly discuss some related works. These are joint works with Jürgen Jost and Lei Liu.

通讯录-几何分析研讨会（武汉大学 2016）

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